

Circuitos Eléctricos



CONVERSIÓN

Δ ↔ Y

Ing. Paulino Aguayo Rojas



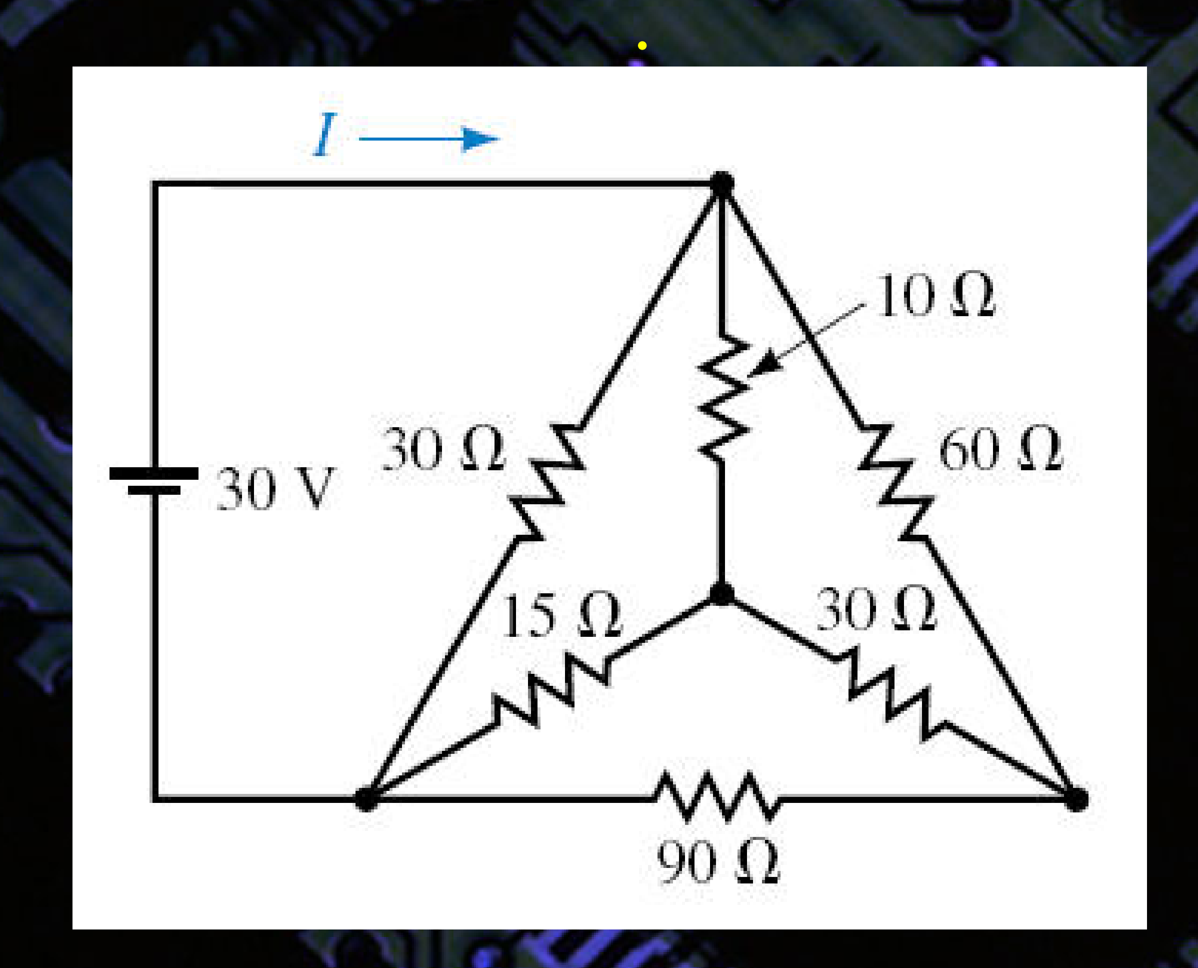
Índice





El problema Y – Δ o T - Π

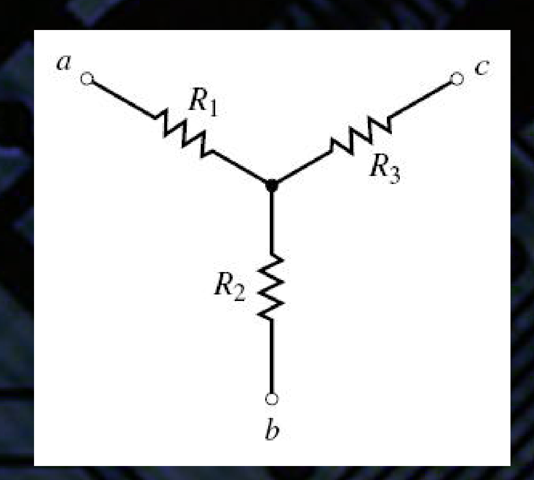


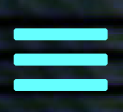


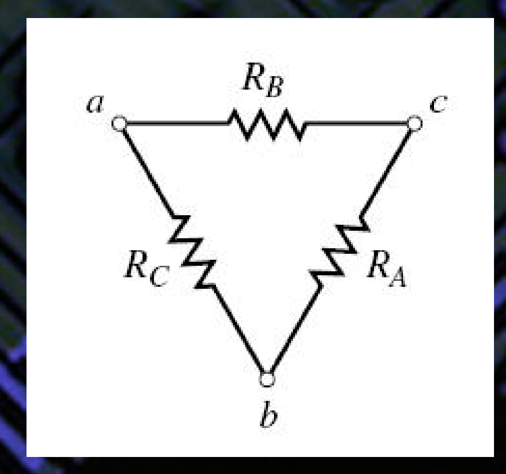


Redes Y – Δ o T – Π equivalentes

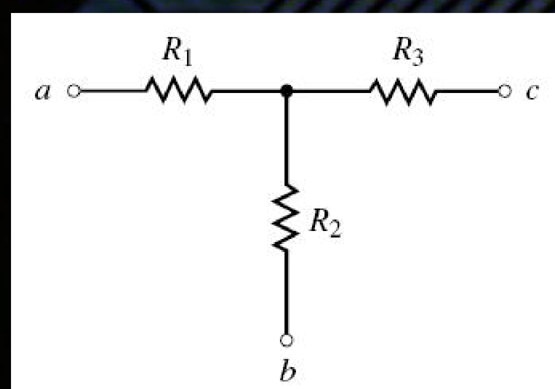


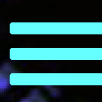


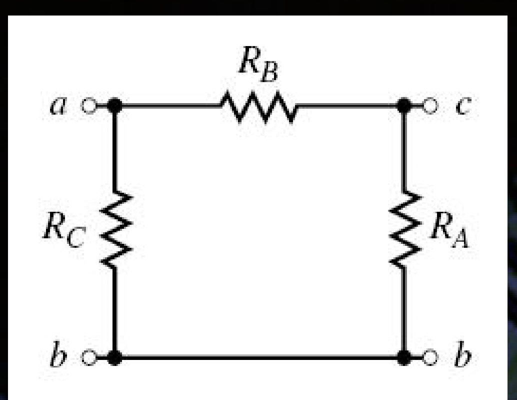








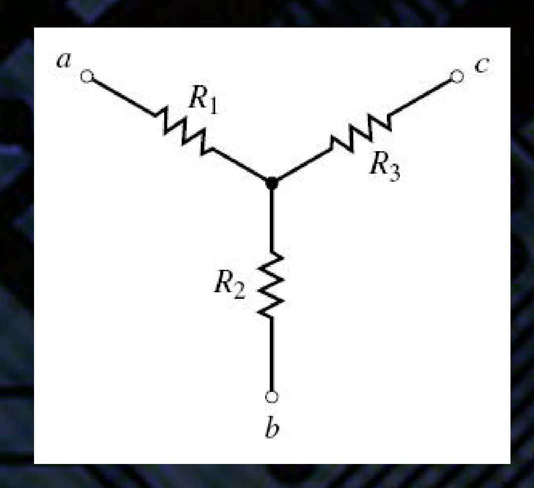




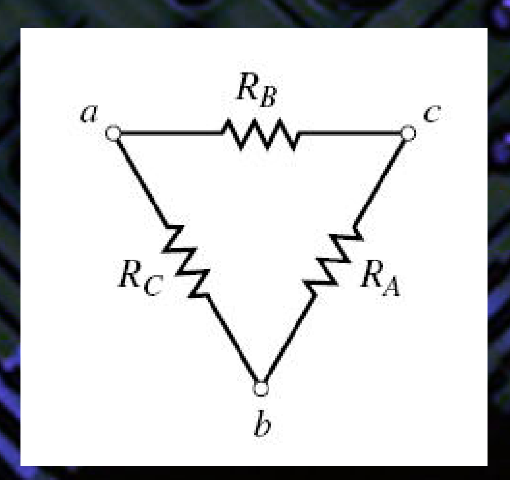


Redes Y – Δ o T – Π equivalentes







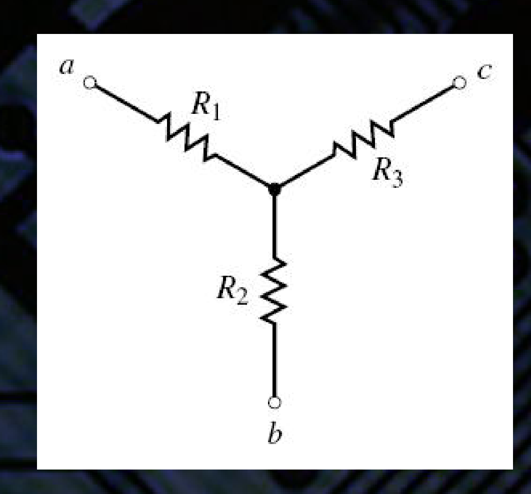


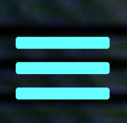
Ambas redes serán equivalentes solo *si la resistencia vista* desde cualquier par de terminales es exactamente la misma.

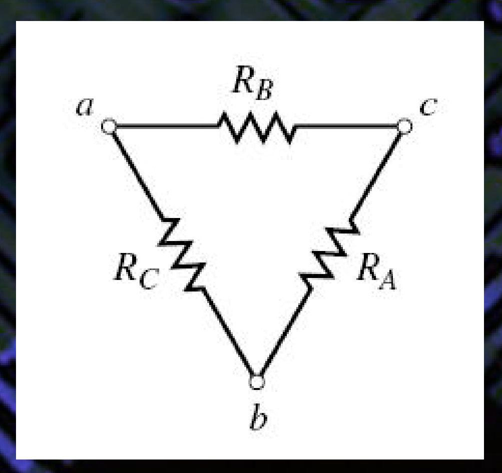


Redes Y – Δ o T – Π equivalentes









Son equivalentes si:

$$R_{ab\text{-}Y} = R_{ab\text{-}\Delta}$$

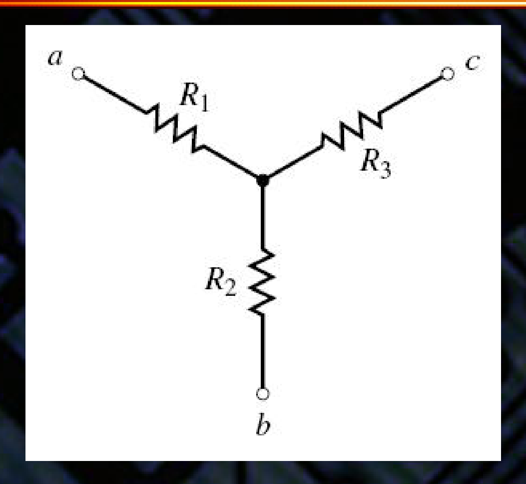
$$R_{ac\text{-}Y} = R_{ac\text{-}\Delta}$$

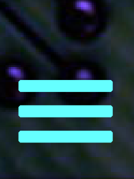
$$R_{bc\text{-}Y} = R_{bc\text{-}\Delta}$$

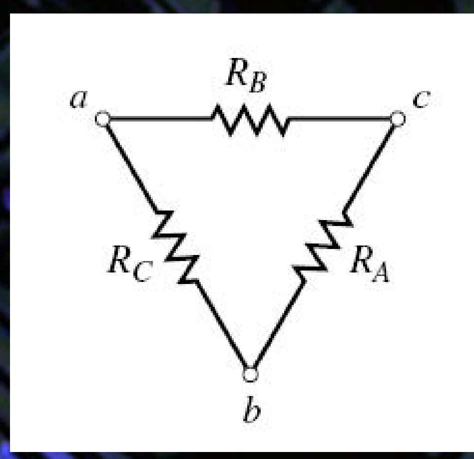


Conversión $\Delta \rightarrow Y$









Vamos a hallar R_1, R_2 y R_3:

$$R_{ab} = R_1 + R_2 \equiv R_{ab} = R_C \| (R_A + R_B)$$

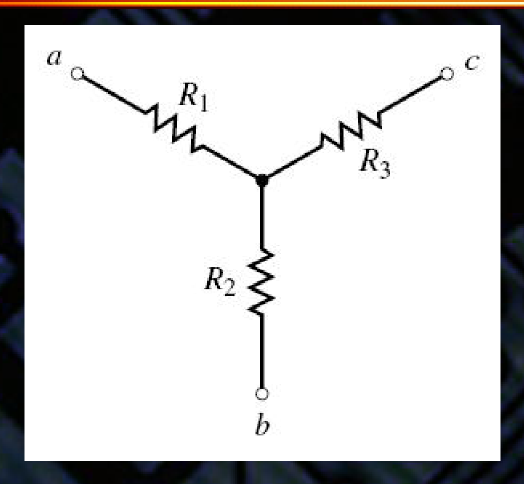
$$R_1 + R_2 = \frac{R_C(R_A + R_B)}{R_A + R_B + R_C}$$

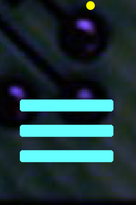
$$R_1 + R_2 = \frac{R_A R_C + R_B R_C}{R_A + R_B + R_C}$$

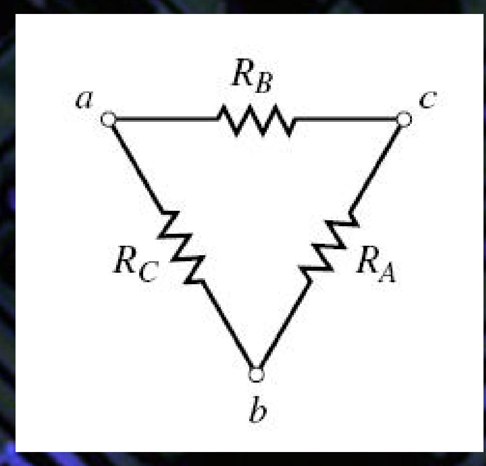




Conversión Δ → Y







Entre los terminales b y c:

$$R_2 + R_3 = \frac{R_A R_B + R_A R_C}{R_A + R_B + R_C}$$

Entre los terminales c y a:

$$R_1 + R_3 = \frac{R_A R_B + R_B R_C}{R_A + R_B + R_C}$$



Conversión Δ → Y



Haciendo:

$$R_1 + R_2 - (R_2 + R_3) = \frac{R_A R_C + R_B R_C}{R_A + R_B + R_C} - \frac{R_A R_B + R_A R_C}{R_A + R_B + R_C}$$

$$R_1 - R_3 = \frac{R_B R_C - R_A R_B}{R_A + R_B + R_C}$$

Luego:

$$R_1 + R_3 + R_1 - R_3 = \frac{R_A R_B + R_B R_C}{R_A + R_B + R_C} + \frac{R_B R_C - R_A R_B}{R_A + R_B + R_C}$$

$$2R_1 = \frac{2R_B R_C}{R_A + R_B + R_C}$$

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$



Conversión Δ → Y - Resultados



$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

Similarmente:

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

Si $R_A = R_B = R_C$

$$R_Y = \frac{R_\Delta}{3}$$



Conversión Y → Δ



De manera similar podemos hacer la conversión inversa

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

$$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

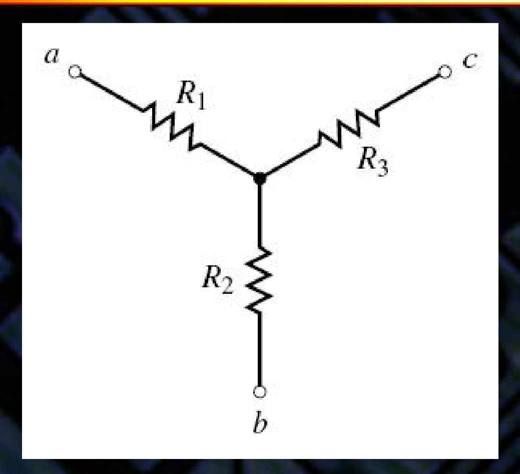
Si $R_1 = R_2 = R_3$

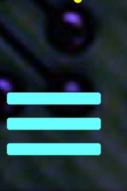
$$R_\Delta = 3R_Y$$

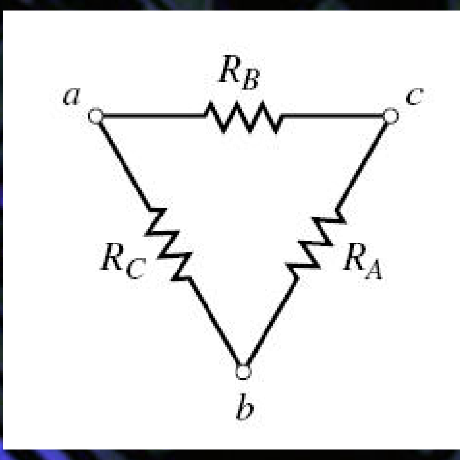




Conversión Δ ↔ Y







$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

$$R_B = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

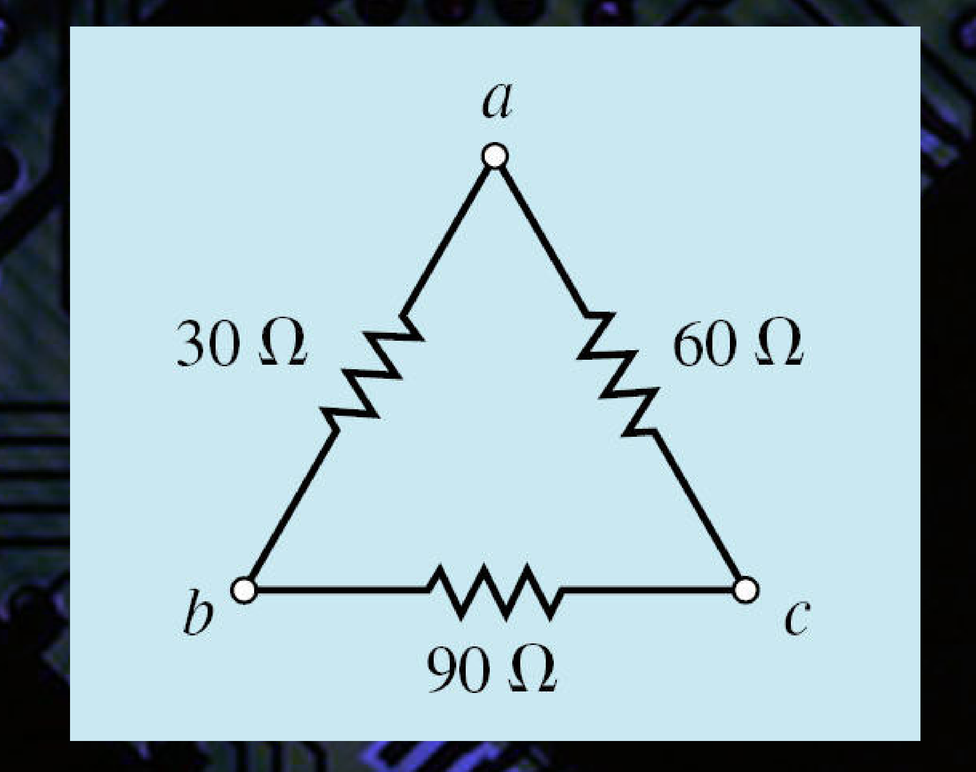
$$R_C = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$



Ejemplo $\Delta \rightarrow Y$



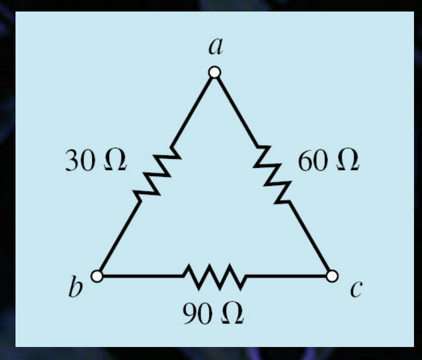
Convertir de $\Delta \rightarrow Y$ el siguiente circuito:

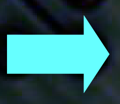




Ejemplo Δ → Y

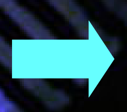






$$R_A = 90\ \Omega$$
$$R_B = 60\ \Omega$$
$$R_C = 30\ \Omega$$

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$
$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$
$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$



$$R_1 = \frac{(30\ \Omega)(60\ \Omega)}{30\ \Omega + 60\ \Omega + 90\ \Omega} = \frac{1800\ \Omega}{180} = 10\ \Omega$$

$$R_2 = \frac{(30\ \Omega)(90\ \Omega)}{30\ \Omega + 60\ \Omega + 90\ \Omega} = \frac{2700\ \Omega}{180} = 15\ \Omega$$

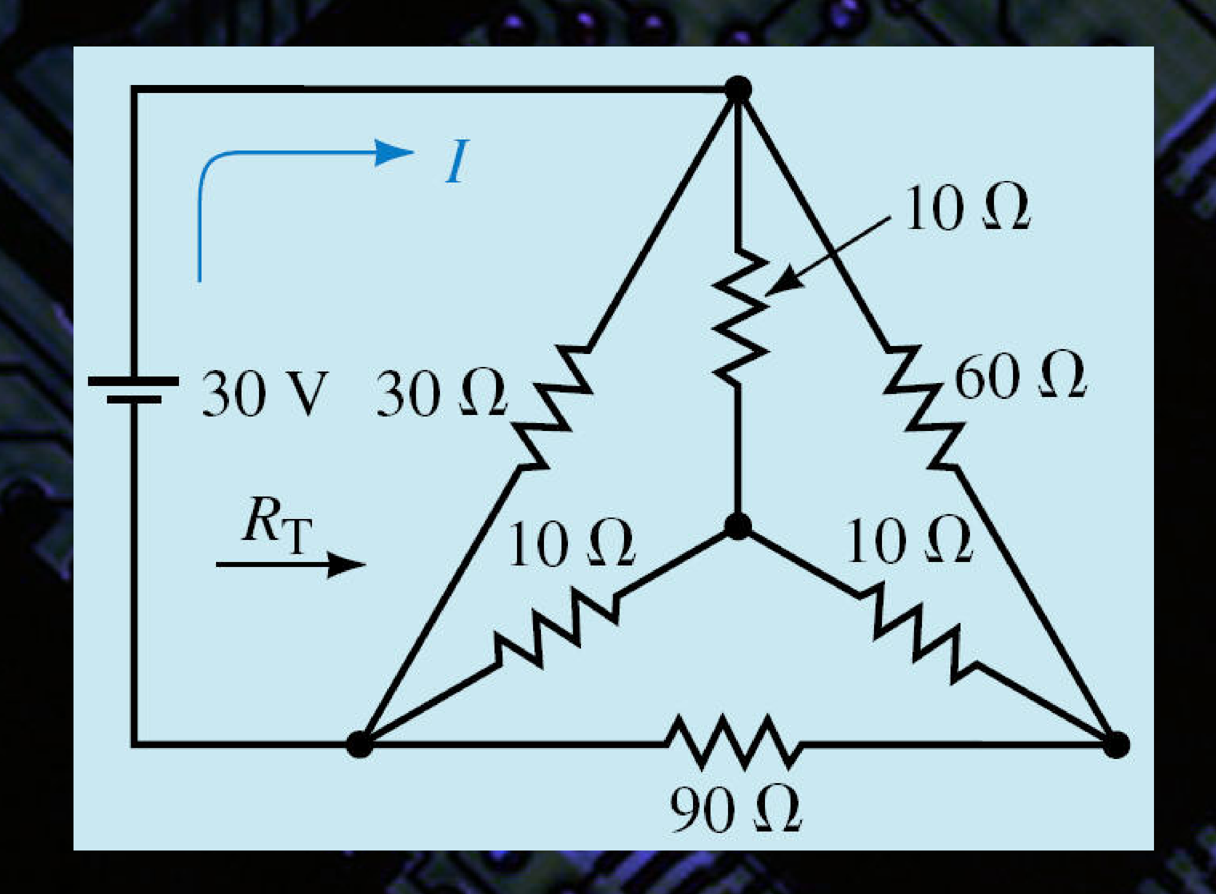
$$R_3 = \frac{(60\ \Omega)(90\ \Omega)}{30\ \Omega + 60\ \Omega + 90\ \Omega} = \frac{5400\ \Omega}{180} = 30\ \Omega$$



Ejemplo 2



Hallar la corriente I y la Rtot para el siguiente circuito:

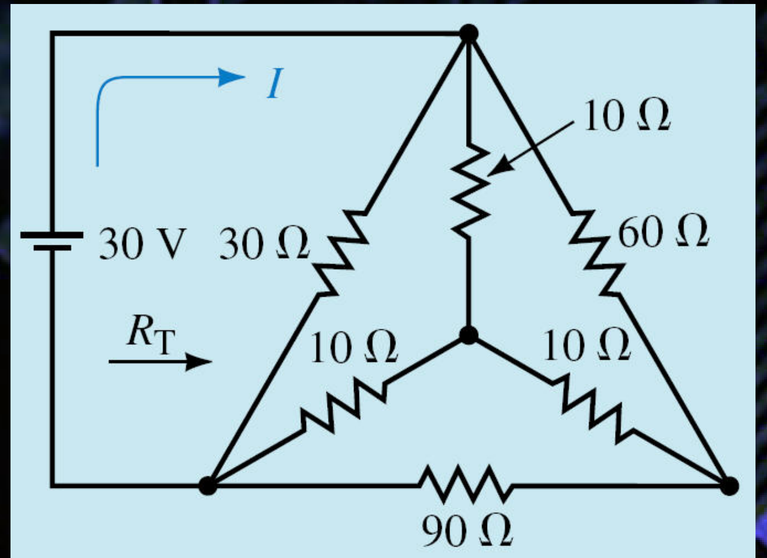


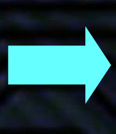


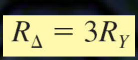
Ejemplo 2

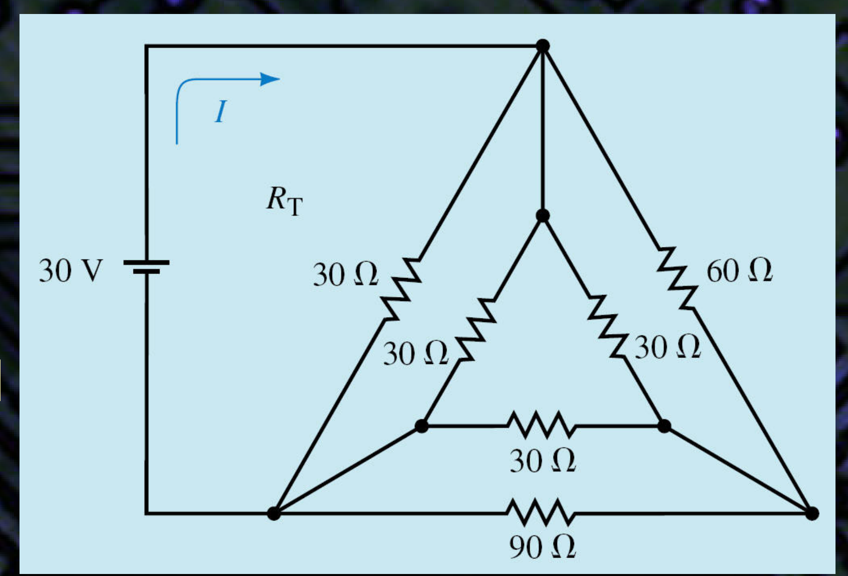


Hallar la corriente I y la Rtot para el siguiente circuito:







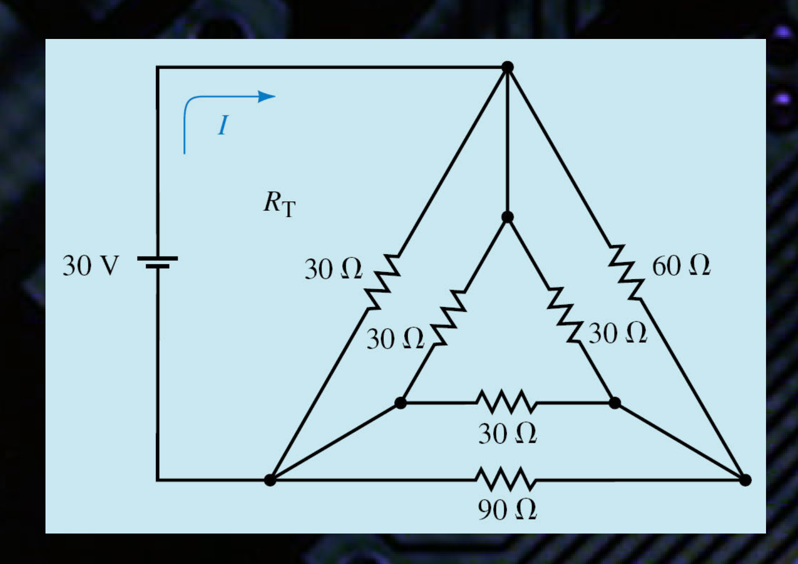


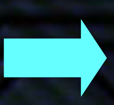


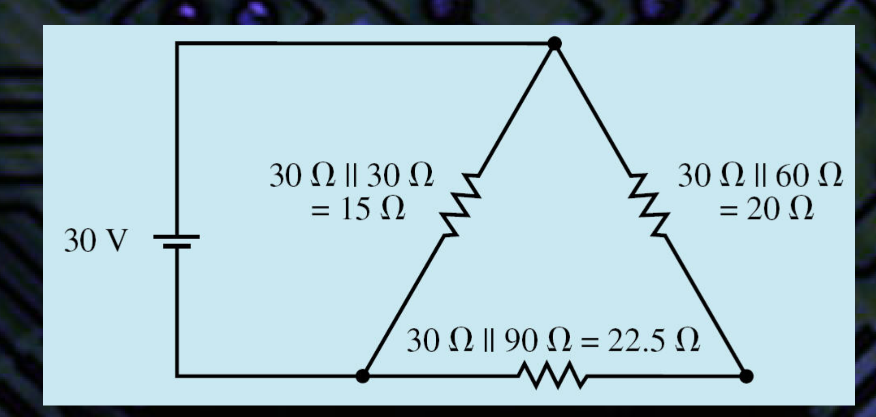
Ejemplo 2



Hallar la corriente I y la Rtot para el siguiente circuito:







$$R_T = 15\ \Omega \| (20\ \Omega + 22.5\ \Omega) = 11.09\ \Omega$$

$$I = \frac{30\ \text{V}}{11.09\ \Omega} = 2.706\ \text{A}$$

FIN



Bibliografía



- **Robbins, Allan y Miller, Wilhem. *Circuit Analysis. 3rd. Edition* Thomson 2003.**
- **Hayt, William, *Análisis de Circuitos para Ingeniería*. 6ª Edición. McGraw Hill, Mexico, 2000.**



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